Frequency analysis of EEG

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9-14 June 2025
Port Harcourt, Nigeria

African Brain Data Network

Outline and topics to cover

Why look at frequencies

What is the spectrum

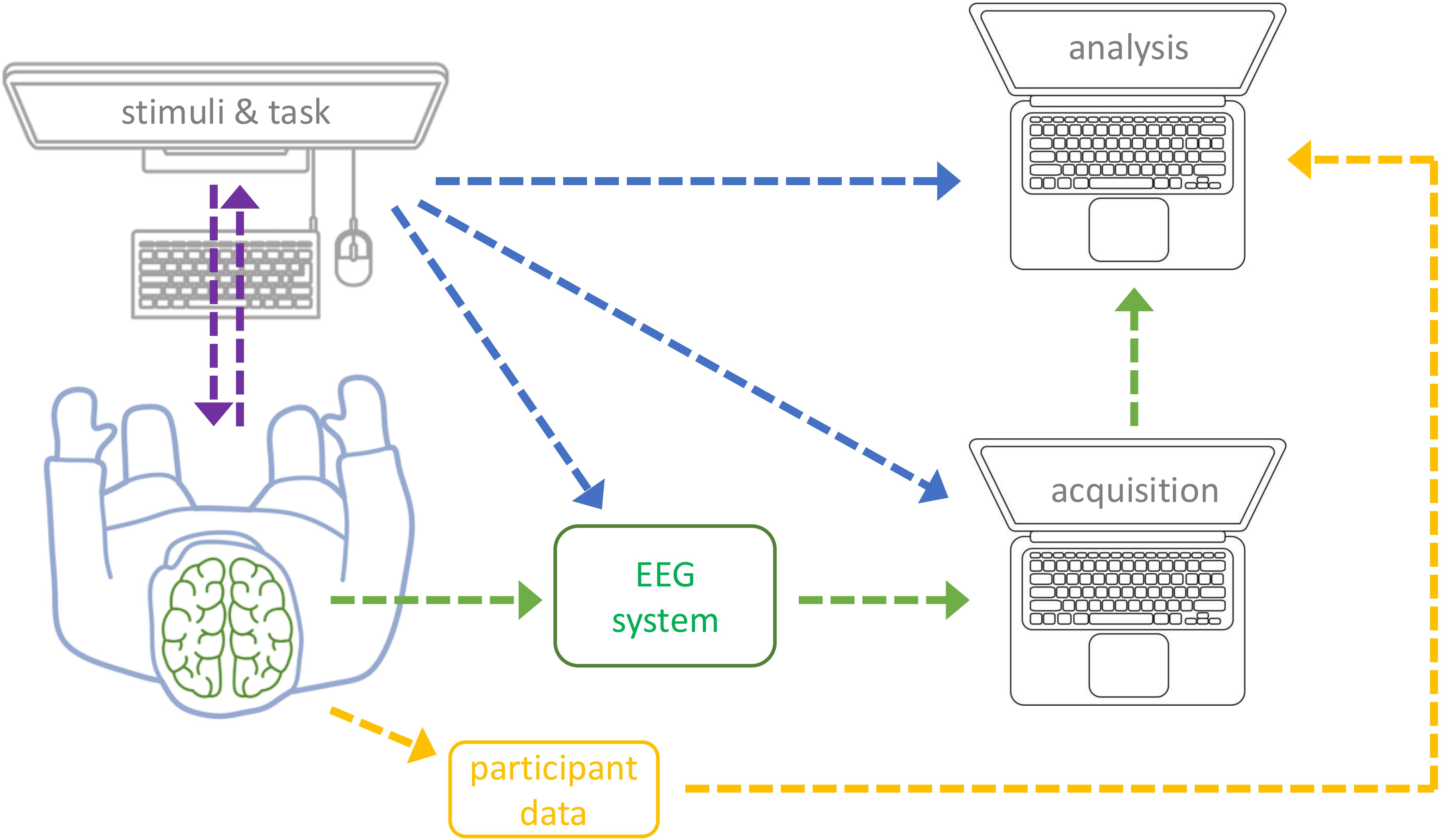
Sine and cosine waves

Fourier analysis

The frequency resolution

Nyquist and the highest frequency

Practical approach


stimuli & task
analysis
acquisition
EEG
system
participant
data

When would you use frequency analysis?

Not all brain activity can be captured in ERPs.

Spontaneous activity shows oscillatory and rhythmic patterns.

EEG frequency analysis makes sense when

- participants are not doing anything specific, aka resting state. That can be eyes-open and/or eyes-closed.
- participants are doing something that is not specifically timed. For example, when watching a movie or listening to continuous speech.

However, when the continuous stream of stimuli can be split into well-timed individual items, then you *can* look at ERPs -> previous lecture.

When you are interested in changes in oscillatory activity around an event, then time-frequency analysis is the appropriate method -> next lecture.

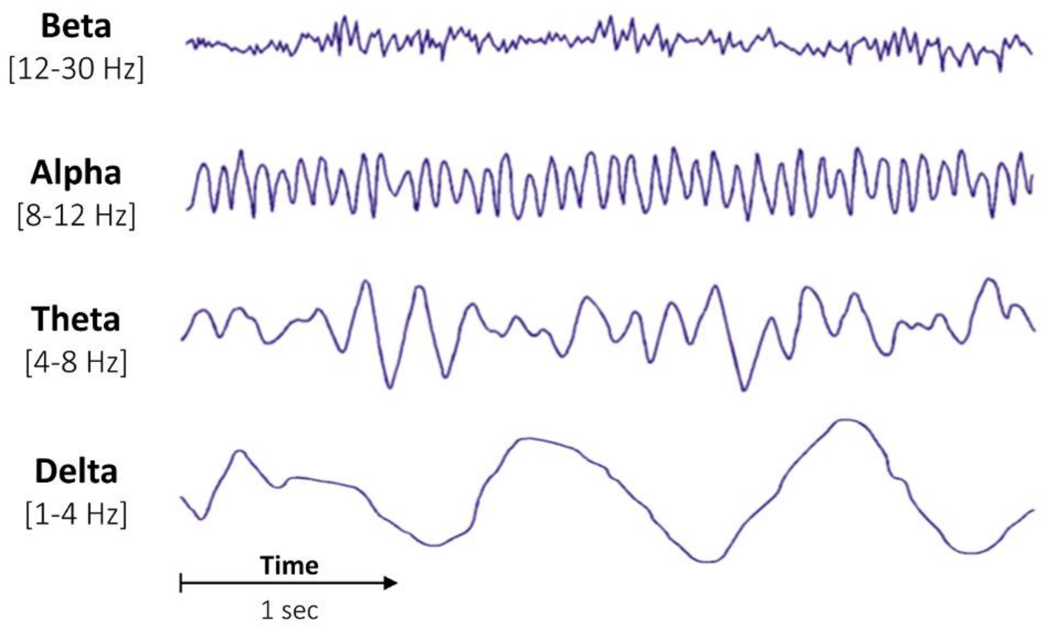
When would you use frequency analysis?

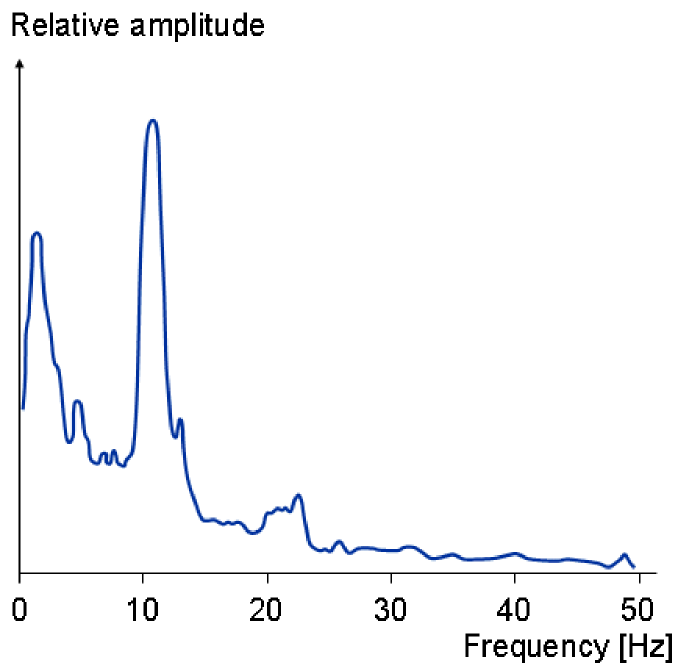
Frequency analysis is often used to compare resting state data in two groups (for example patients and healthy control participants), or to compare two experimental states (for example before and after treatment).

Frequency analysis is not so often used in cognitive experimental paradigms with well-defined stimuli and responses.

Frequency or spectral or Fourier analysis

Fourier analysis is a math trick that takes a complicated wave (like sound, or EEG) and breaks it down into its basic ingredients—pure sine waves of different pitches and volumes.





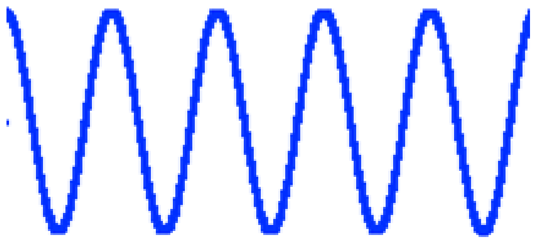
Spectral analysis

Deconstructing a time-domain signal into its components, using simple oscillatory functions: cosines and sines.

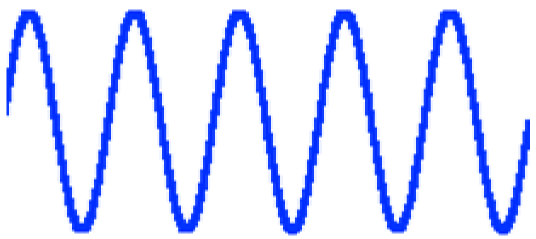
When it goes up and down *once* per second, it is a 1 Hz signal

When it goes up and down *twice* per second, it is a 2 Hz signal

When it goes up and down *once per ½ second*, it is a 2 Hz signal.

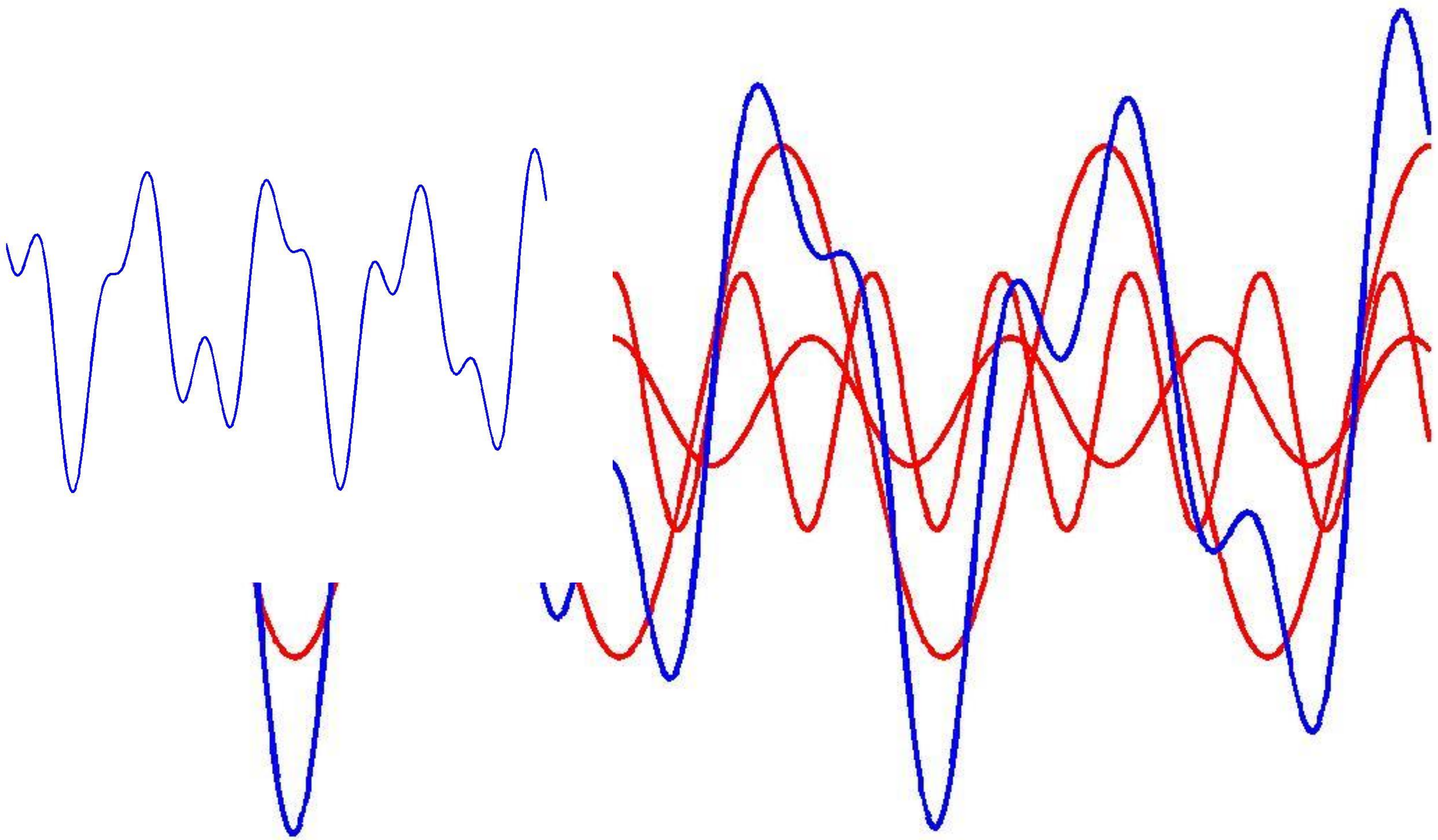


The frequency is one-divided-by the time that it takes to go up and down.

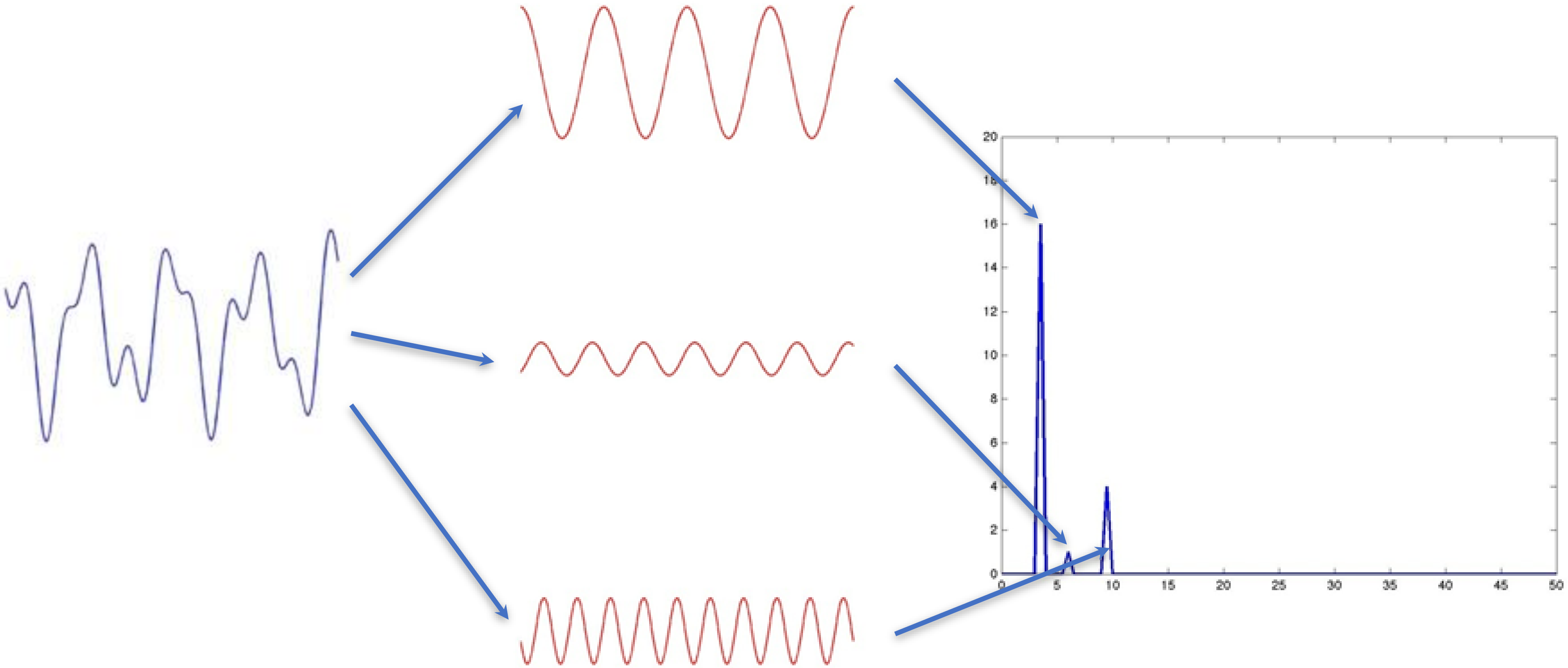


$F = 1/T$

Spectral decomposition: the principle

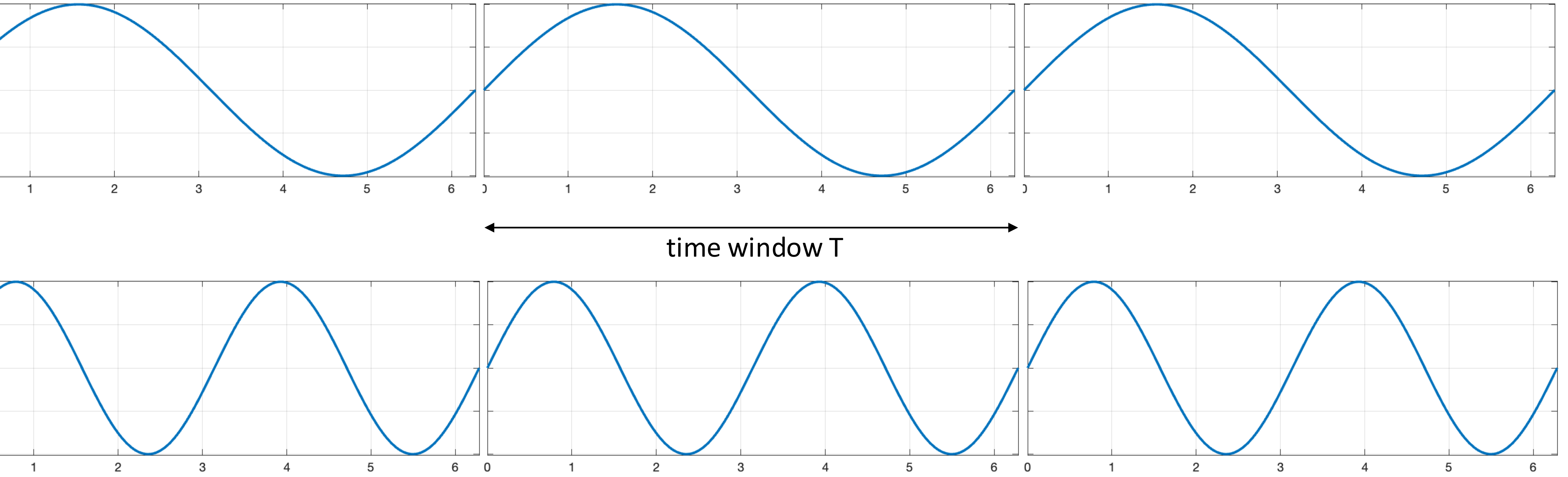


Spectral decomposition: the power spectrum

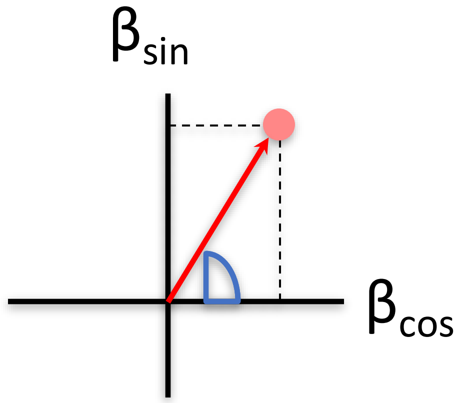


Discrete Fourier analysis

- Even though we only observe the signal in a short window, we assume that it is continuous and periodic.
- This implies that we have sine (or cosine) functions with an integer multiple of oscillations in the window: 1 cycle, 2 cycles, 3 cycles, …



Spectral analysis is similar to GLM

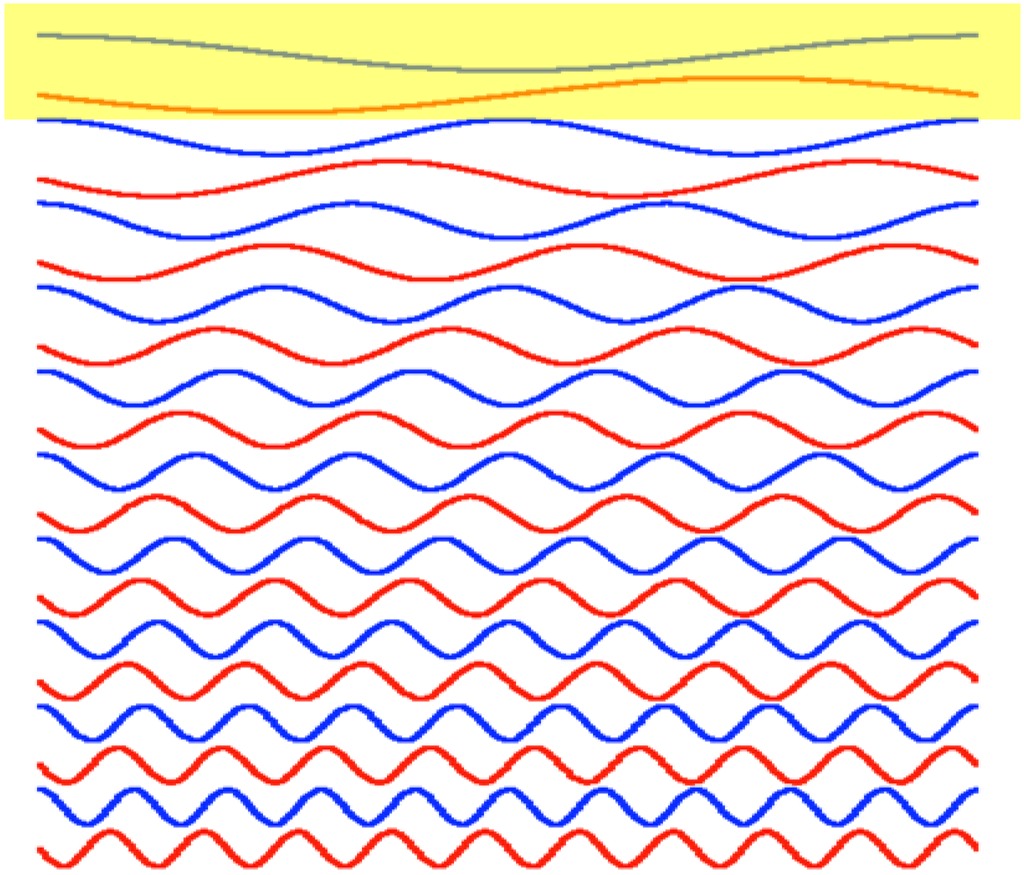


$Y = \beta * X$

X set of (orthogonal) basis functions

βi contribution of basis function i to the data.

β for the cosine and sine at a given frequency map onto the amplitude and phase.



Going from N time points to N cosine/sine components

Each cosine/sine pair reflects 1 frequency bin so ~N/2 frequencies can be estimated

Frequencies correspond to integer number of cycles of basis functions in time window

Frequency resolution

The cosine and sine waves correspond to “basis functions”

The basis functions have an integer number of cycles in the time window being analyzed and are orthogonal.

One-second window:

T = 1 s -> frequency resolution of 1 Hz

F = 1 Hz

F = 2 Hz

F = 3 Hz

…

Frequency resolution

The cosine and sine waves correspond to "basis functions"

The basis functions have an integer number of cycles in the time window being analyzed and are orthogonal.

Two-second window:

T = 2 s -> frequency resolution of ½ or 0.5 Hz

F = 0.5 Hz

F = 1.0 Hz

F = 1.5 Hz

…

Frequency resolution

The cosine and sine waves correspond to “basis functions”

The basis functions have an integer number of cycles in the time window being analyzed and are orthogonal.

Ten-second window:

T = 10 s -> frequency resolution of 1/10 or 0.1 Hz

F = 0.1 Hz

F = 0.2 Hz

F = 0.3 Hz

…

Frequency resolution

When we have a long window we get a very high frequency resolution. That might be useful if you listen to the radio (which has very sharp frequencies), but not when you listen to the brain.

Physiological oscillations like alpha (around 10 Hz) fluctuate a bit.

No reason to estimate the spectral content at a higher resolution than those fluctuations.

Computing the power spectrum

Following EEG preprocessing, we cut the long recording into short segments of length, estimate the power spectrum for each segment, and average those.

The averaging of the power spectrum makes it much more robust and less sensitive to noise.

The average of the noise in ERPs goes towards zero.

Power is never zero but always positive, so the average power does not go to zero.

Nyquist frequency

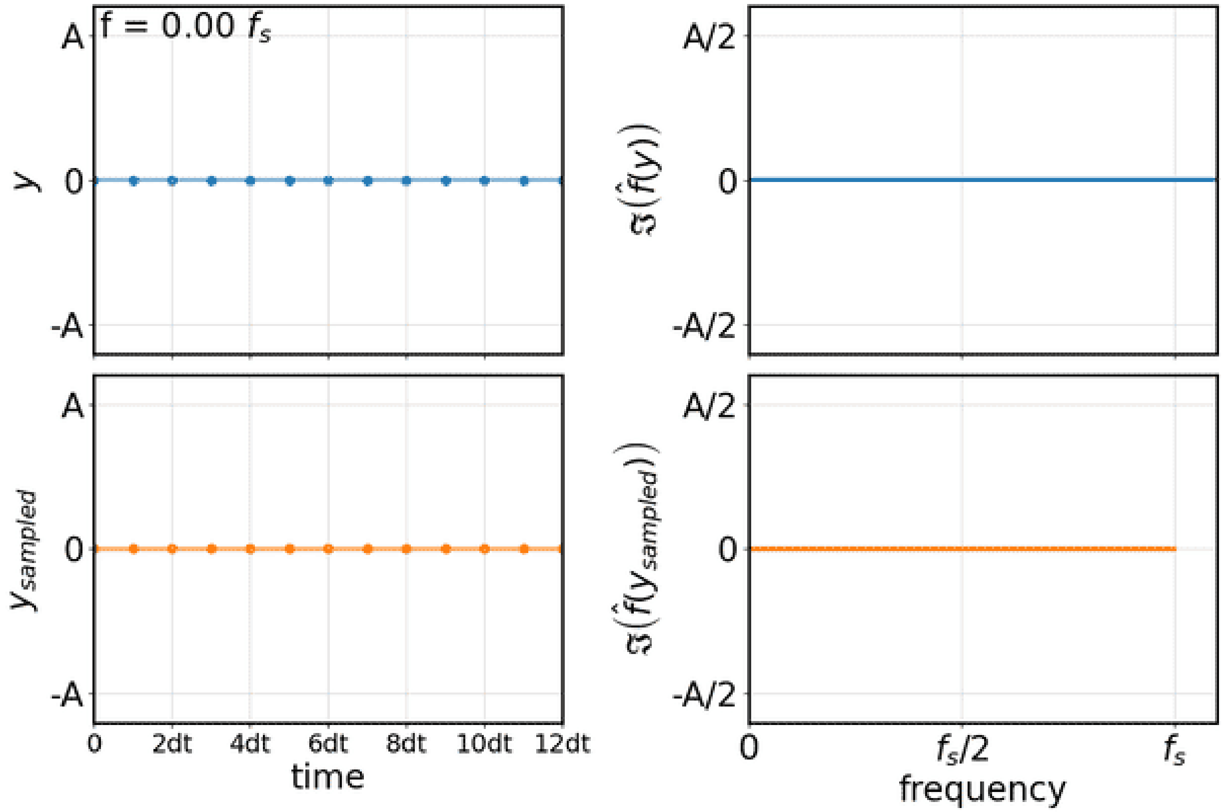
The highest frequency that can be resolved depends on the sampling frequency or the time Δt between two samples.

Fsample = 1/Δt
Nyquist frequency = 1/(2*Δt) = Fsample/2

If you sample at 250 Hz, the highest frequency you can estimate is 125 Hz.

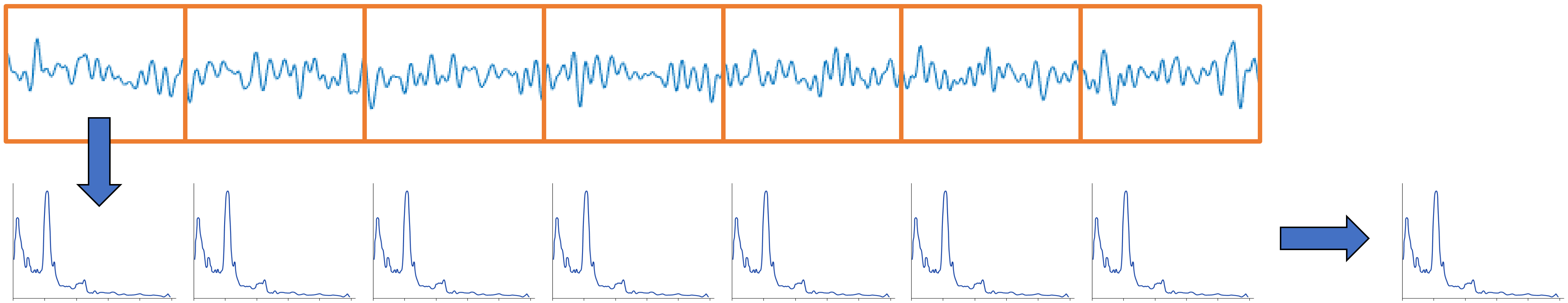
Frequencies present in the analog signal prior to sampling will show up as noise at lower frequencies.

EEG hardware often uses an “anti aliasing” filter. So in practice you don’t see signal up to 1/2, but only up to 1/3 or 1/4 of the sampling frequency.

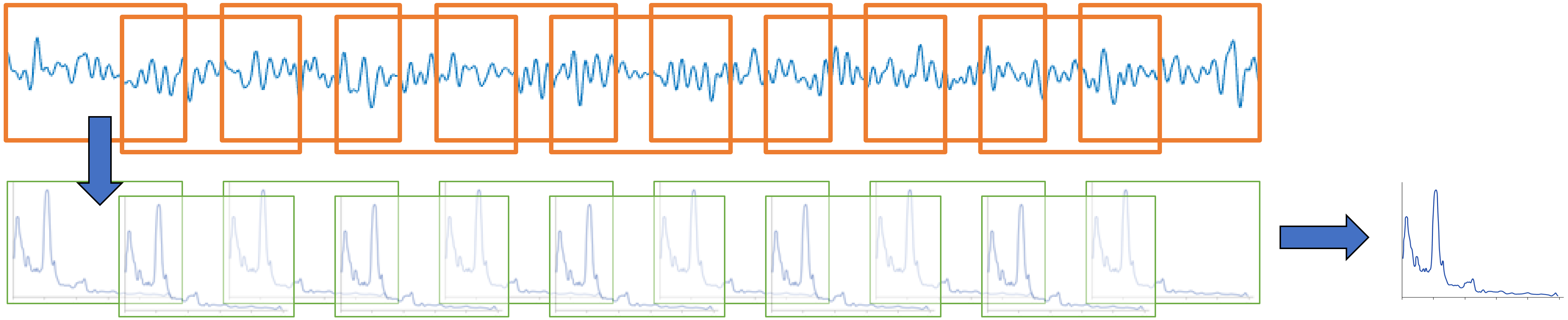


https://en.wikipedia.org/wiki/Aliasing

Segment, FFT, average



Segment with some overlap



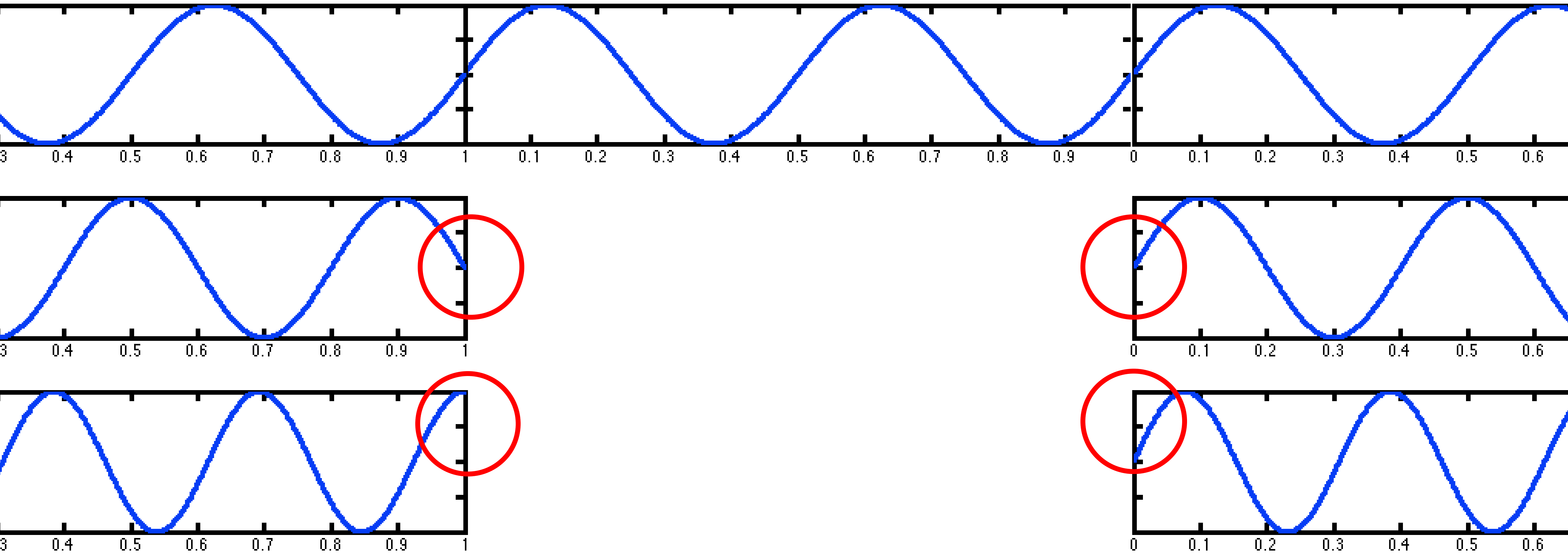
Spectral leakage and tapering

Since we assume that the signal in each segment is continuous (over segments) and periodic, a signal that "jumps" from the end to the start needs a lot of frequencies.

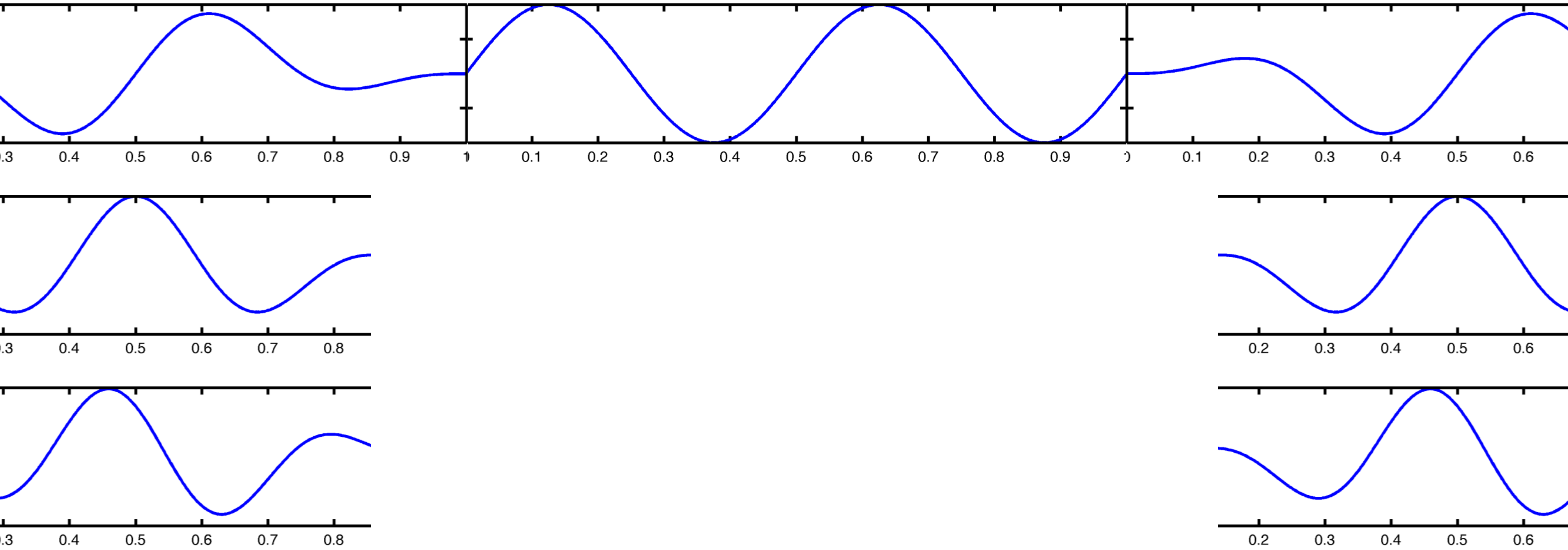
The effect of intermediate frequencies appearing at other frequencies is called "spectral leakage".

Tapering or windowing is a method to reduce spectral leakage.

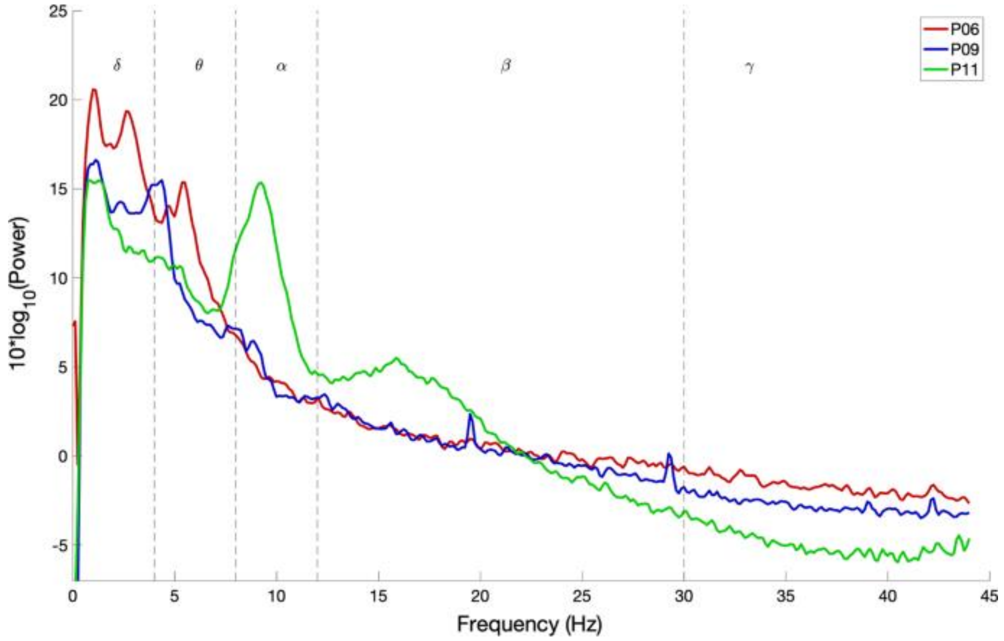
Tapering in spectral analysis



Tapering in spectral analysis



Frequency analysis recipe



If you don't know when in time something happens.

If you assume that the brain activity is stationary.

Take a long segment in which the EEG assumed to be stationary. For example, from 10 seconds after closing the eyes, up to the moment that the experimenter gives the instruction to open the eyes.

Segment that long time window into shorter windows of 1 or 2 seconds. These windows can be overlapping. Apply a taper to each window, compute the FFT to get the power spectrum for each short windows, and average those.

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