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Forward and inverse modelling of MEG

Jan-Mathijs Schoffelen

Donders Institute, Radboud University, Nijmegen, NL

janmathijs.schoffelen@donders.ru.nl

Overview

Motivation and background

Forward modeling

- Source model

- Volume conductor model

- Sensor model

Inverse modeling - biophysical models

- Single and multiple dipole fitting

- Distributed source models

- Beamforming methods

Inverse modeling - independent components

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Motivation 1

Strong points of EEG and MEG

- Temporal resolution (~ 1 ms)

- Characterize individual components of ERP

- Oscillatory activity

- Disentangle dynamics of cortical networks

Weak points of EEG and MEG

- Measurement on outside of brain

- Overlap of components

- Low spatial resolution

Motivation 2

If you find an ERF/TF component, you may want to characterize it in physiological terms

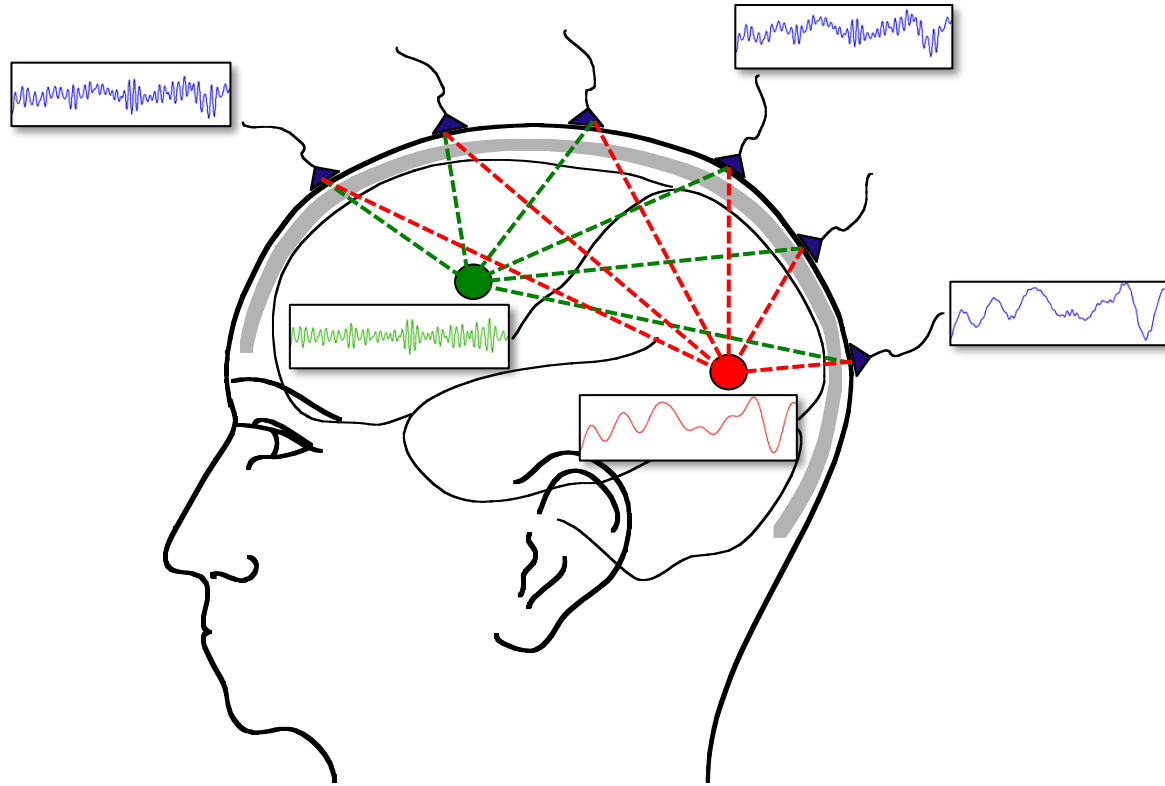
- Time or frequency are the “natural” characteristics

- “Location” requires interpretation of the scalp topography

Forward and inverse modeling helps to interpret the topography

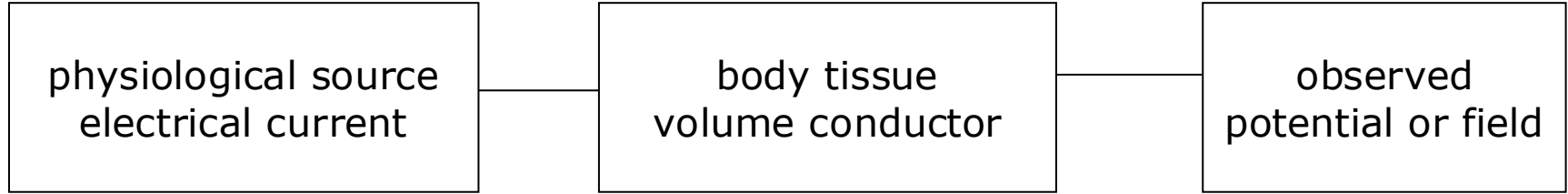
Forward and inverse modeling helps to disentangle overlapping source timeseries

Superposition of source activity



Biophysical source modelling: overview

forward model



inverse model

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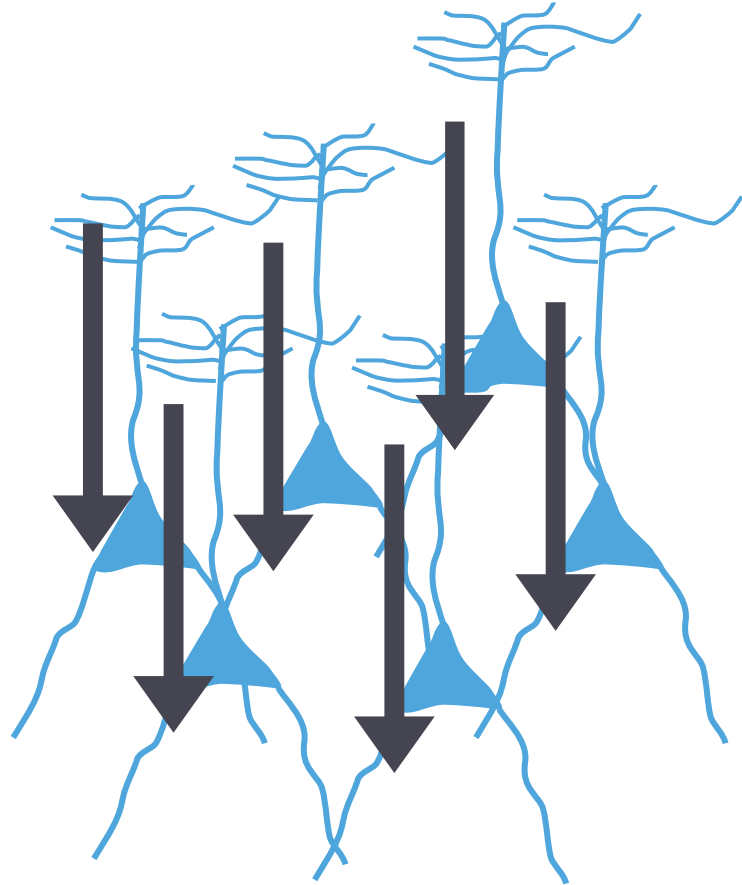
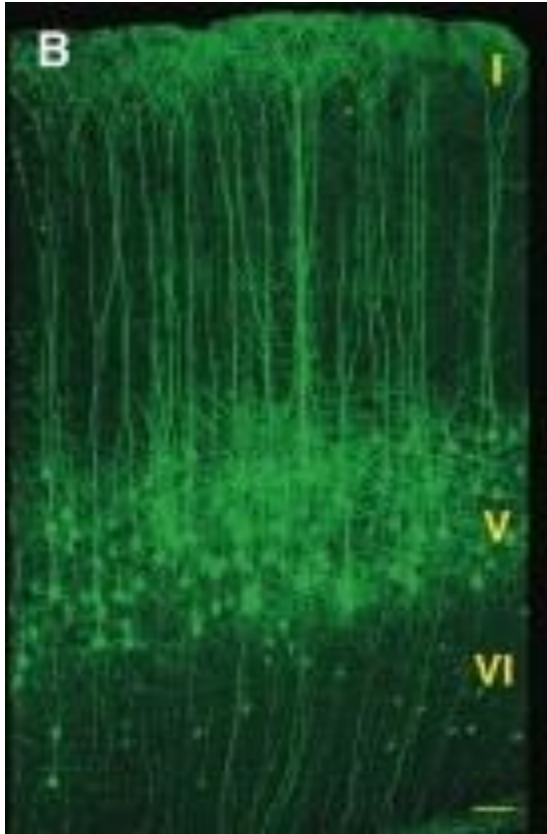
Distributed source models

Beamforming methods

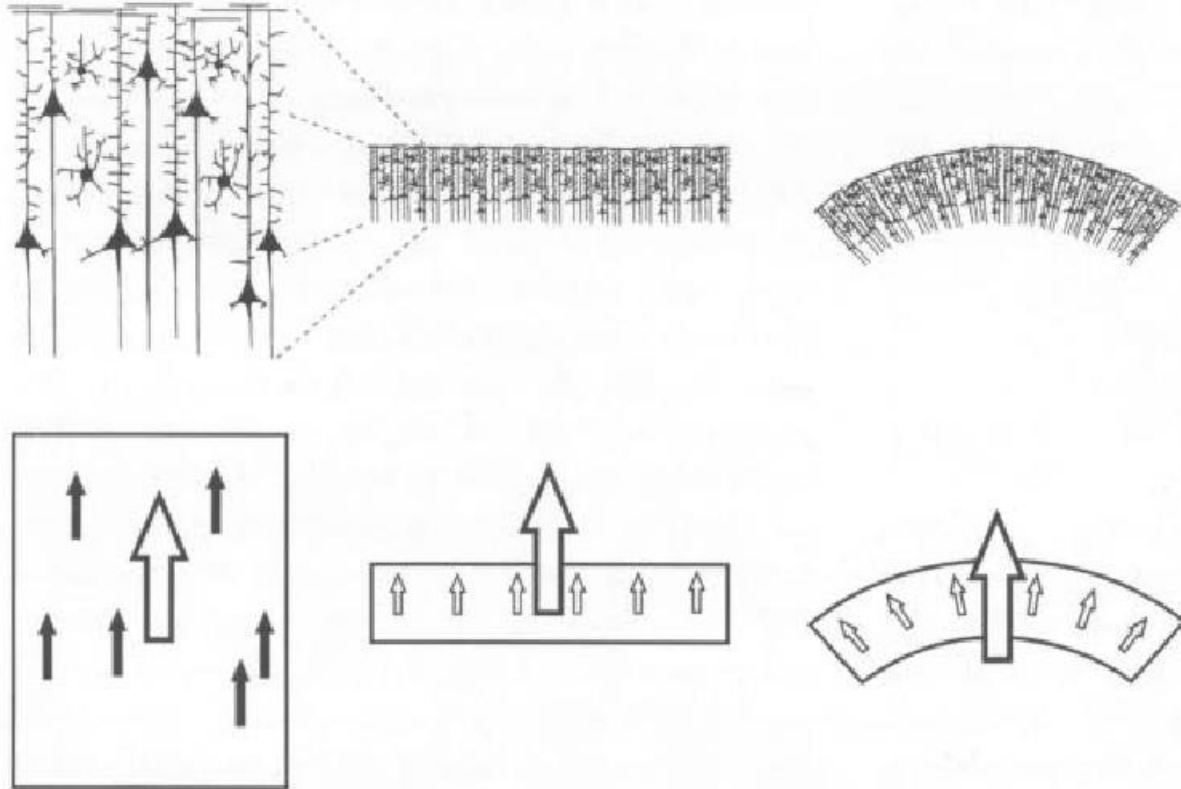
Inverse modeling - independent components

Summary

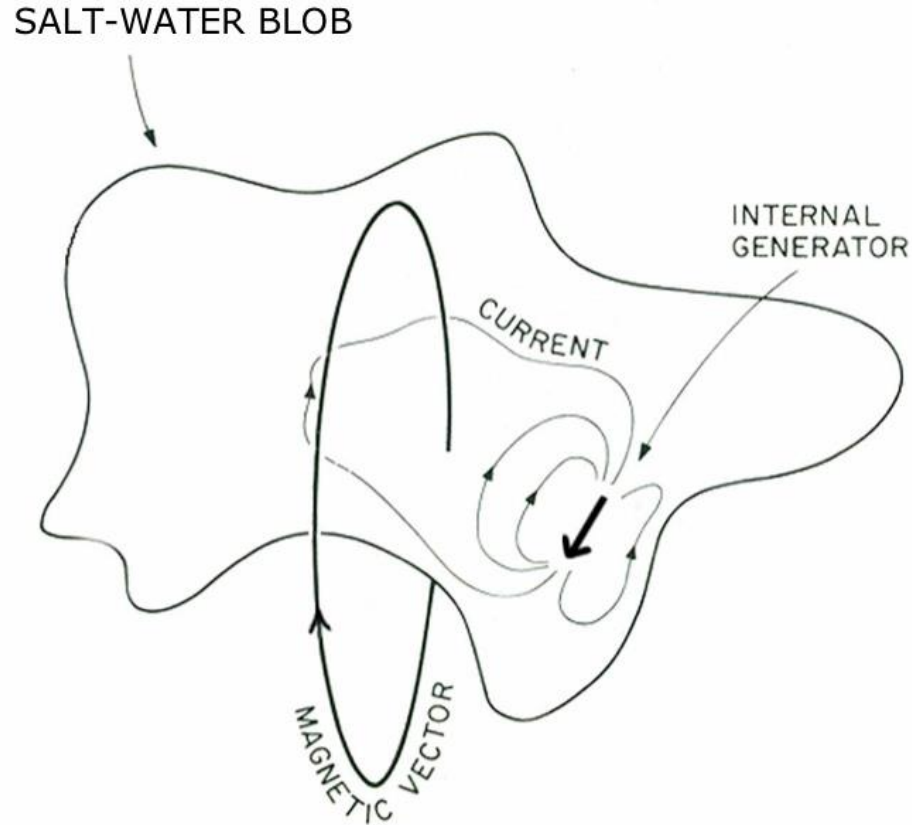
What produces the electric current



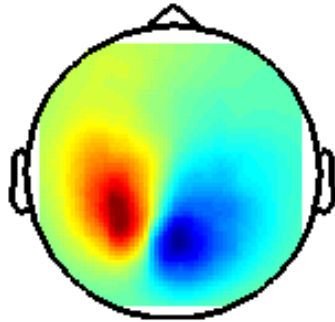
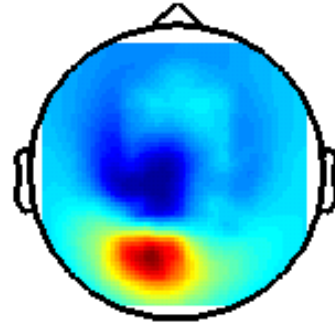
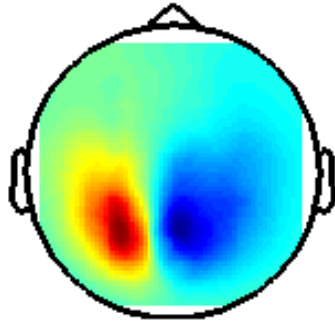
Equivalent current dipoles



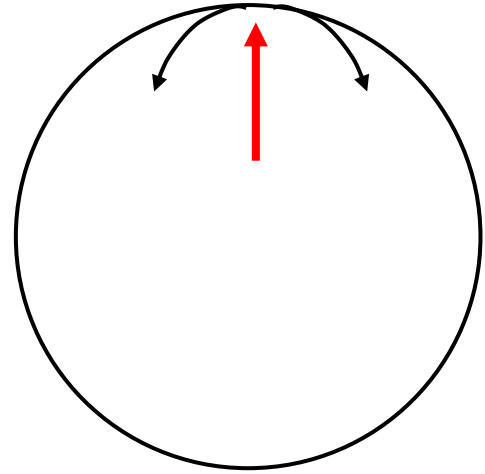
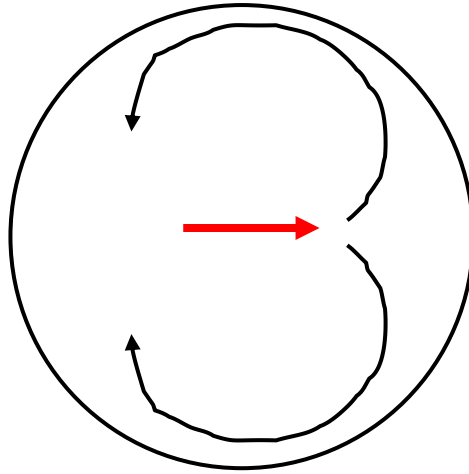
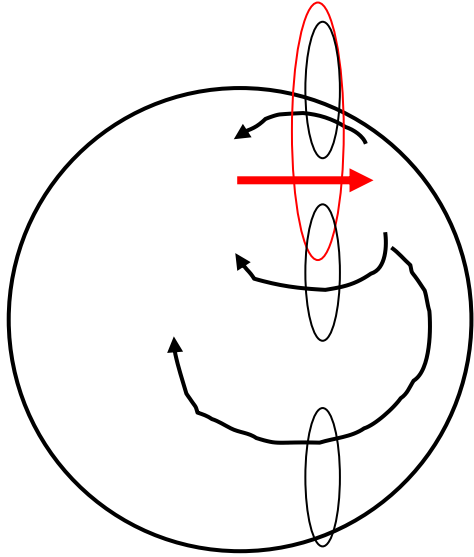
Currents produce magnetic fields



Currents produce magnetic fields



Currents produce magnetic fields



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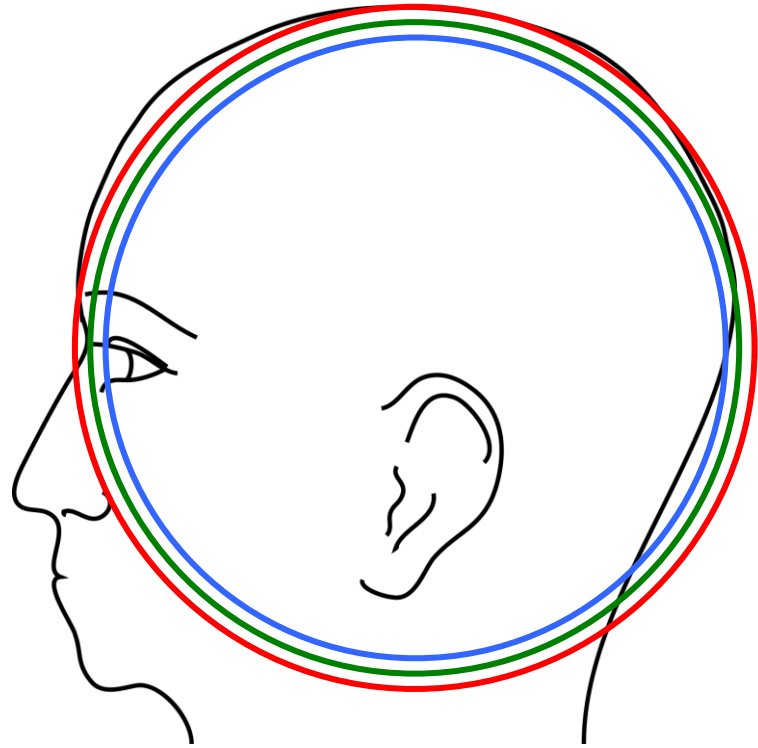
Volume conductor

describes electrical properties of tissue

describes geometrical model of the head

describes **how** the currents can flow, not where they originate from

same volume conductor for EEG as for MEG, but also for tDCS, tACS, TMS, ...



Volume conductor

Analytical methods

single/multiple sphere models

Computational methods for volume conduction
problem that allow for realistic geometries

Guido Nolte's 'singleshell' method

BEM *Boundary Element Method*

FEM *Finite Element Method*

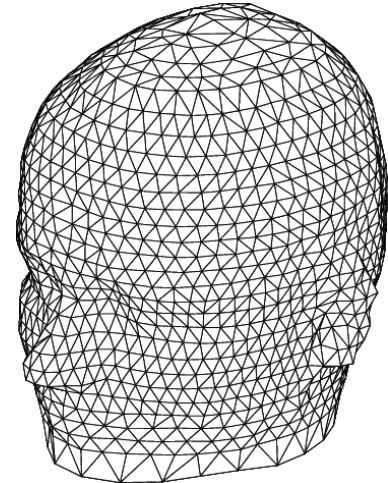
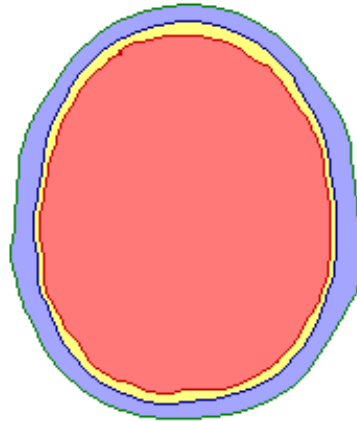
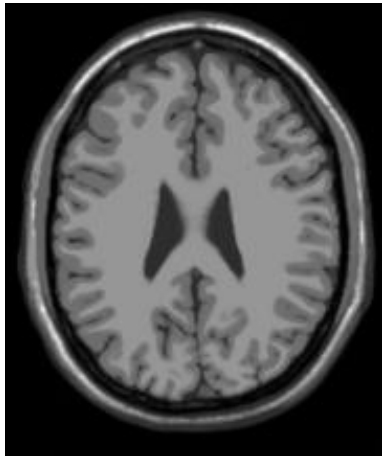
BEM/singleshell/FEM require anatomical processing

BEM: segmentation into different tissue types

extract surface descriptions (triangulations)

assumes homogeneous conductivity per tissue compartment

singleshell: surface description of inner skull surface



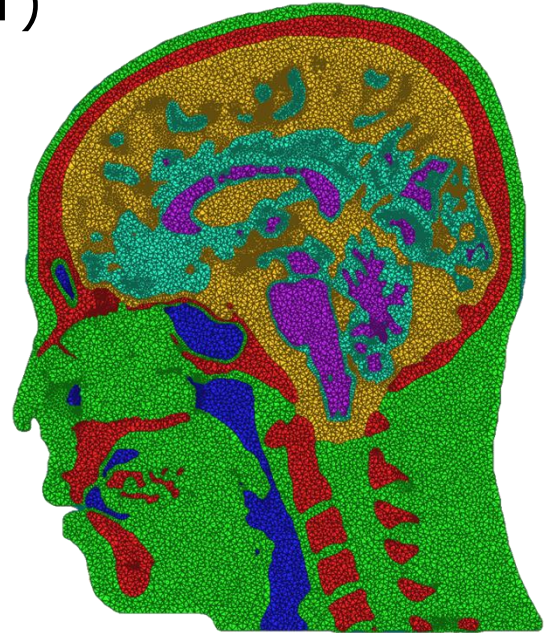
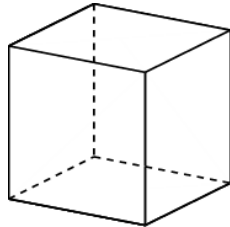
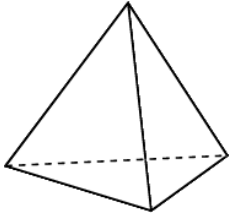
BEM/singleshell/FEM require anatomical processing

FEM: tessellation of 3D volume in hexahedral/tetrahedral elements

conductivity per tissue type, volume element, or even anisotropic within tissue (white matter)

computationally expensive

creation of geometries is not trivial



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Sensor model

Location of sensors relative to head

Orientation of sensors relative to head

Coregistration

Calibration

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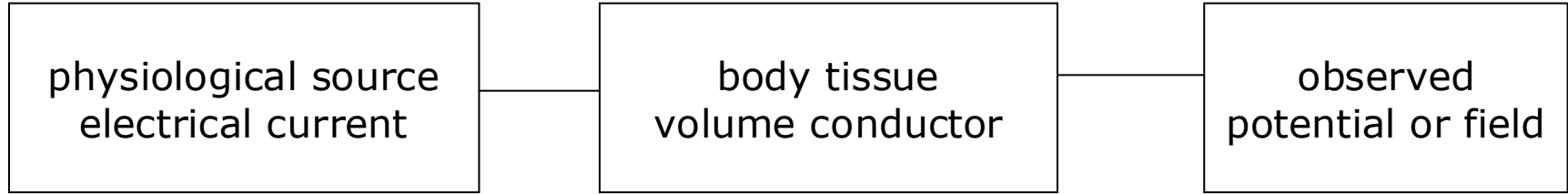
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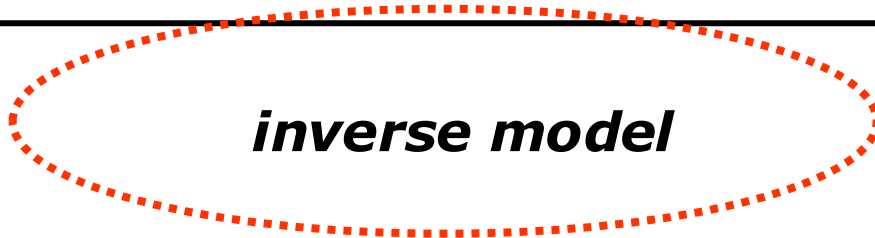
Summary

Biophysical source modelling: overview

forward model



inverse model



Inverse localization: demo



Inverse modelling

The measurement gives us the magnetic field distributions over time

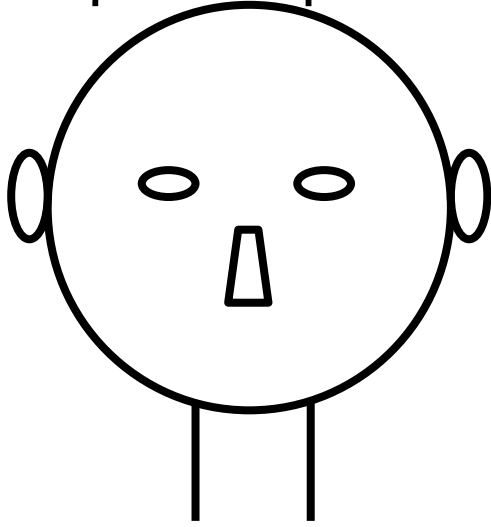
With the forward model we can compute the magnetic field distribution at the sensors, given a modeled source

With the inverse model we model the sources that give rise to the measured data

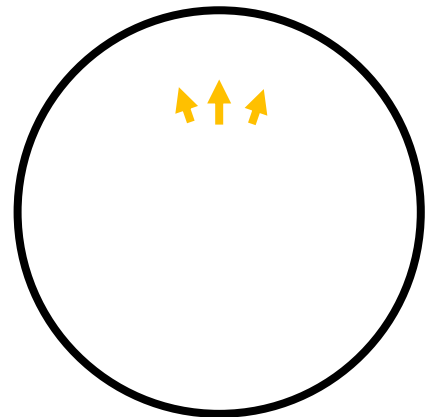
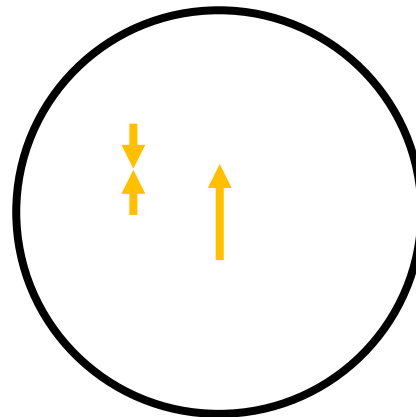
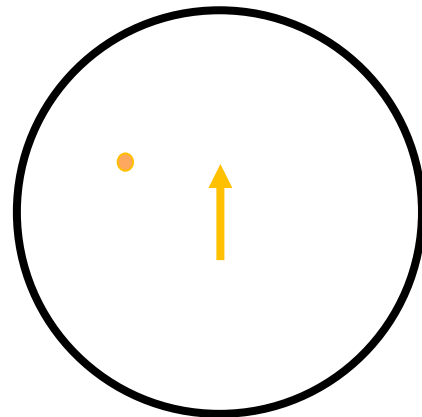
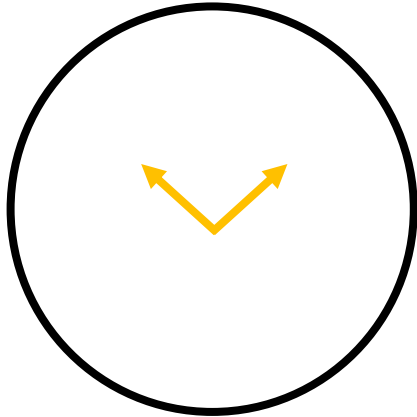
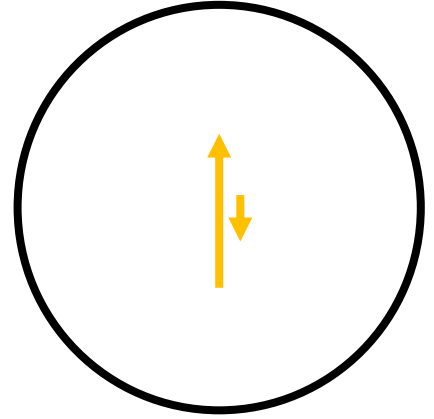
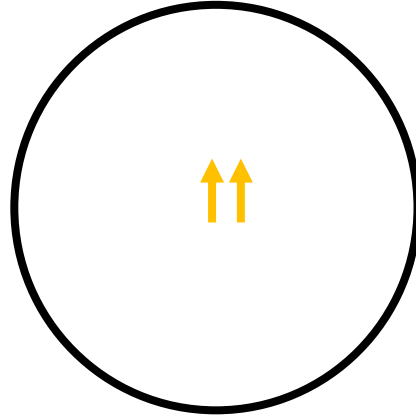
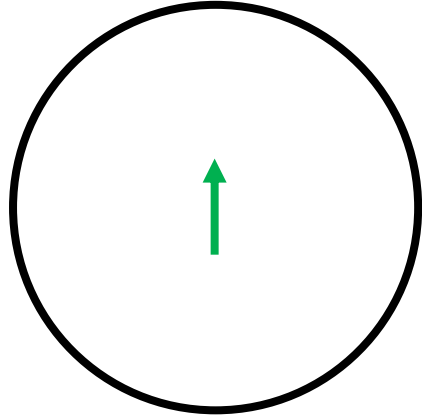
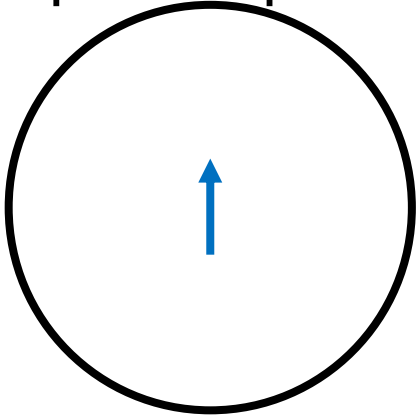
The inverse solution to map the measured MEG onto the source model is “ill-posed”

The (discrete) number of sensors is much smaller than the (infinite) number of possible source combinations

Ill-posed problem: Infinite number of solutions



Ill-posed problem: Infinite number of solutions



Inverse methods: additional constraints needed

Single and multiple dipole models - underdetermined

Minimize error between model and measured potential/field

Distributed source models - overdetermined

Will give perfect fit of model to the measured potential/field

Additional constraint on source smoothness, power or amplitude

Spatial filtering

Scan the whole brain with a single dipole and compute the filter output at every location

Beamforming (e.g. LCMV, SAM, DICS)

Multiple Signal Classification (MUSIC)

Estimating source timecourse activity

$$Y = G_1 X_1 + G_2 X_2 + \dots + G_n X_n + \text{noise}$$

few sources

distributed sources

one at a time

$$X'(t) = W Y(t)$$

dipole fitting

minimum norm estimate

beamforming

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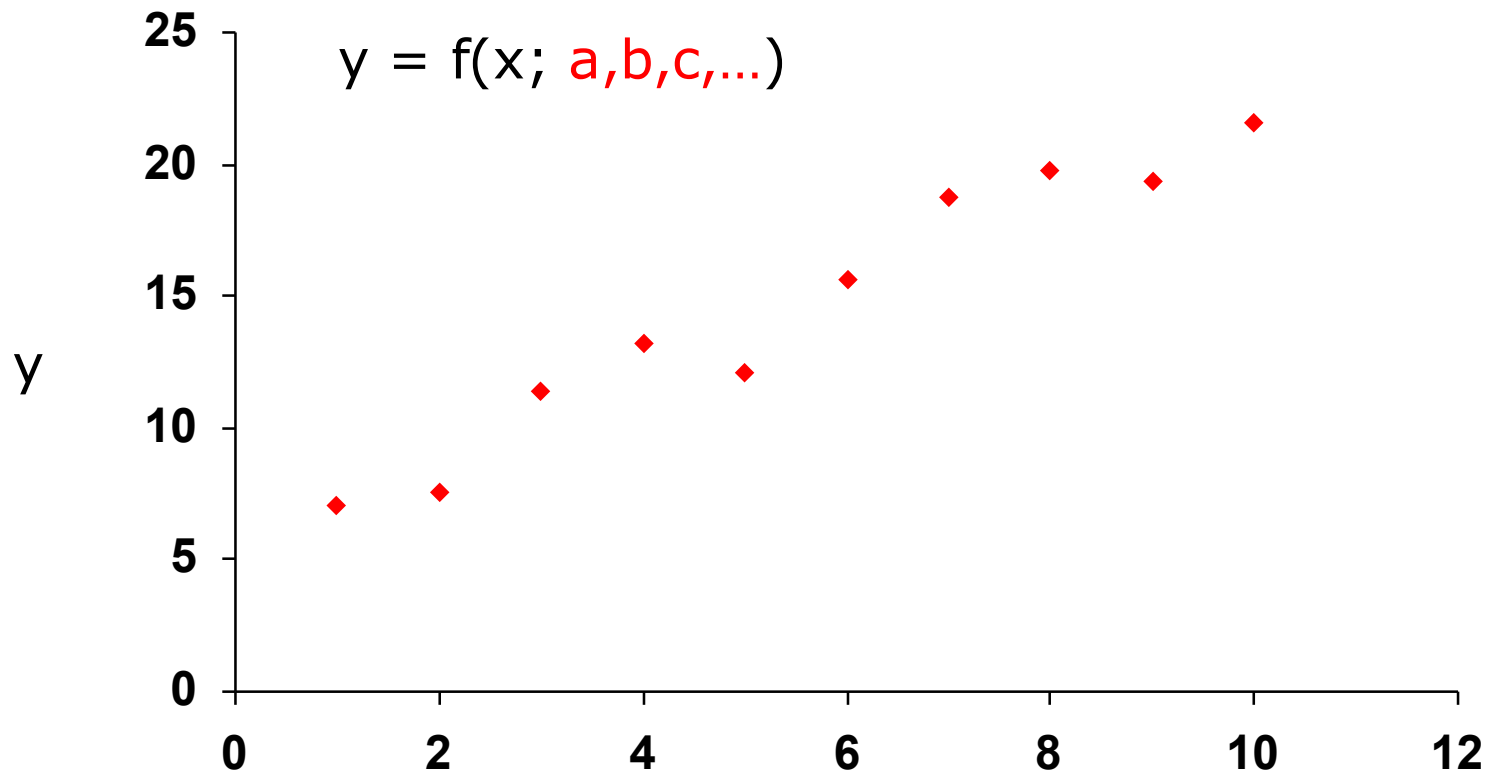
- Distributed source models

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Inverse modeling - independent components

Summary

Single or multiple dipole models - Parameter estimation



Parameter estimation: dipole parameters

source model with
few parameters

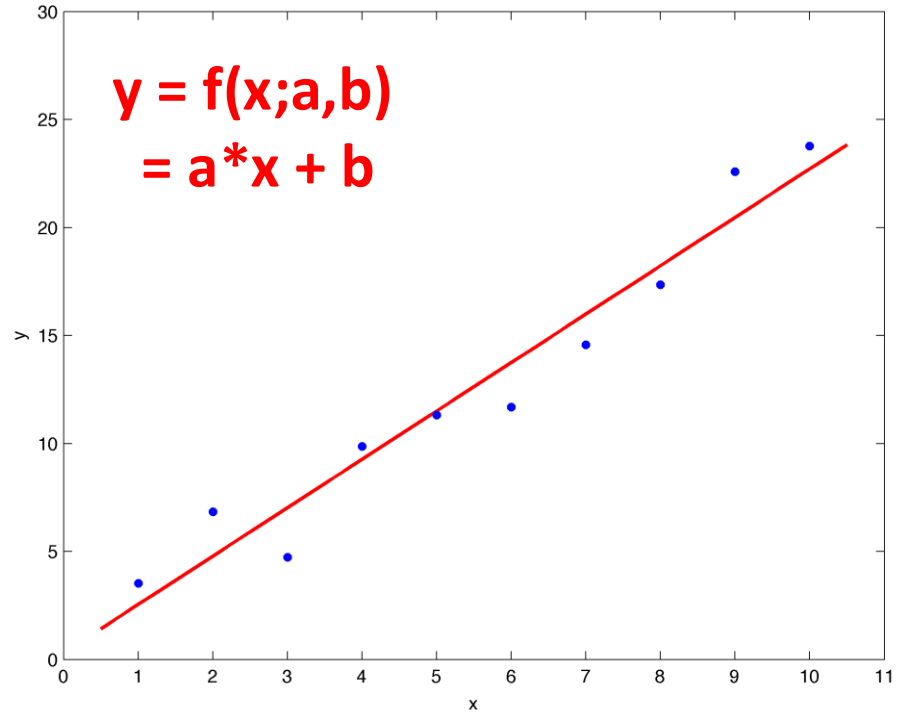
position

orientation

strength

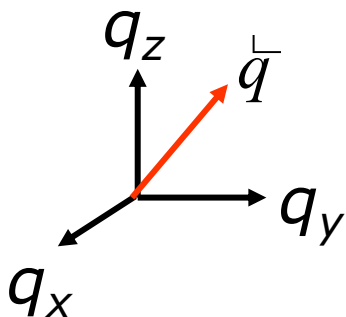
compute the model data

minimize difference
between actual and model
data



Linear parameters: linear estimation (math = 'simple')

$$Y = G_x q_x + G_y q_y + G_z q_z = \begin{bmatrix} G_{x,1} & G_{y,1} & G_{z,1} \\ G_{x,2} & G_{y,2} & G_{z,2} \\ \vdots & \vdots & \vdots \\ G_{x,N} & G_{y,N} & G_{z,N} \end{bmatrix} \cdot \begin{bmatrix} q_x \\ q_y \\ q_z \end{bmatrix} = \mathbf{G} \cdot \vec{q}$$



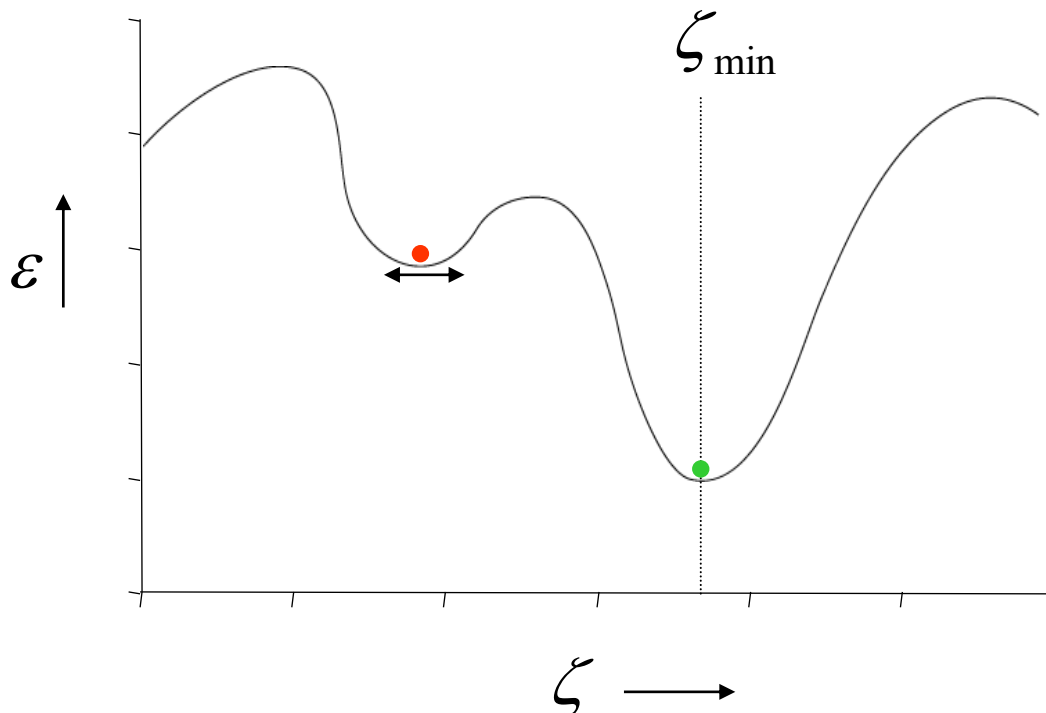
$$\begin{aligned} Y &= \mathbf{G} \cdot \vec{q} \\ &= \mathbf{G}(\xi) \cdot \vec{q} \end{aligned}$$

$$\vec{q} = \mathbf{G}^{-1} \cdot Y$$

Non-linear parameters

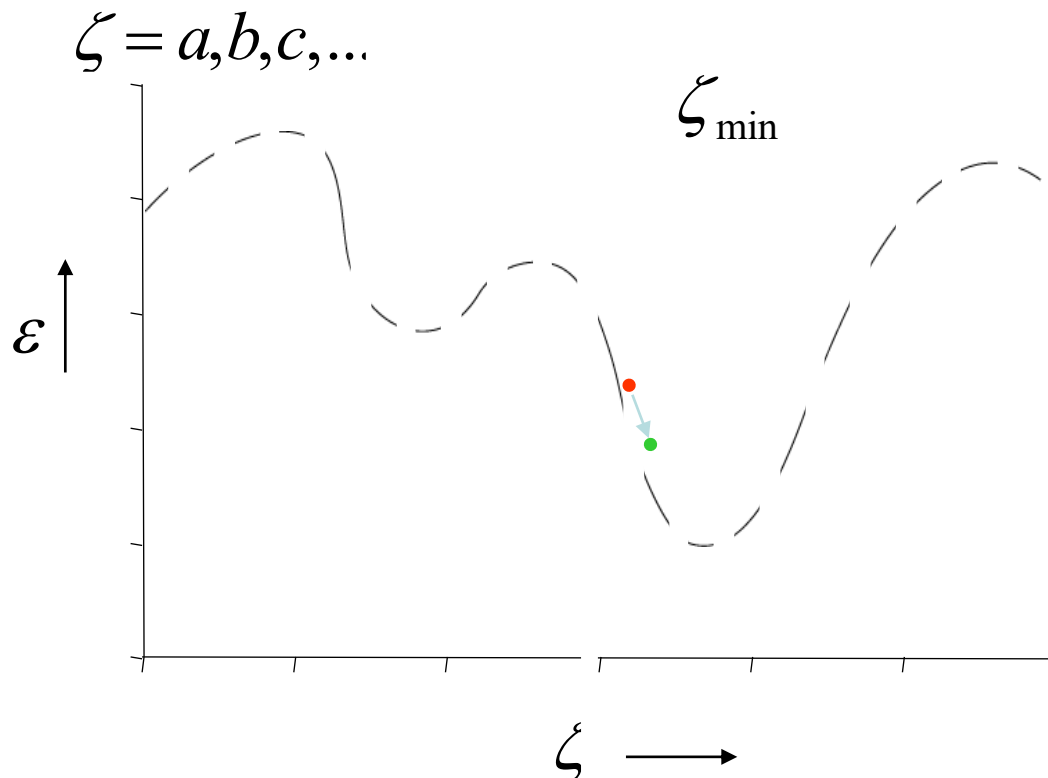
$$error(\zeta) = \sum_{i=1}^N (Y_i(\zeta) - V_i)^2 \Rightarrow \min_{\zeta} (error(\zeta))$$

$\zeta = a, b, c, \dots$



Non-linear parameters: gradient descent optimization

$$\text{error}(\zeta) = \sum_{i=1}^N (Y_i(\zeta) - V_i)^2 \Rightarrow \min_{\zeta} (\text{error}(\zeta))$$



Non-linear parameters: grid search

One dimension, e.g. location along medial-lateral

100 possible locations

Two dimensions, e.g. med-lat + inf-sup

$100 \times 100 = 10.000$

Three dimensions

$100 \times 100 \times 100 = 1.000.000 = 10^6$

Two dipoles, each with three dimensions

$100 \times 100 \times 100 \times 100 \times 100 \times 100 = 10^{12}$

Single or multiple dipole models - Strategies

Single dipole:

scan the whole brain, followed by iterative optimization

Two dipoles:

scan with symmetric pair, use that as starting point for iterative optimization

More dipoles:

sequential dipole fitting

Sequential dipole fitting to explain spread of activity

Assume that activity starts “small”

- explain earliest ERP component with single equivalent current dipole

Assume later activity to be more widespread

- add ECDs to explain later ERP components
- estimate position of new dipoles
- re-estimate the activity of all dipoles

Iterative and interactive (hence subjective) process

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Distributed source model

Position of the source is **not estimated** as such

Pre-defined grid (3D volume or cortical sheet)

Strength is estimated

In principle easy to solve, however...

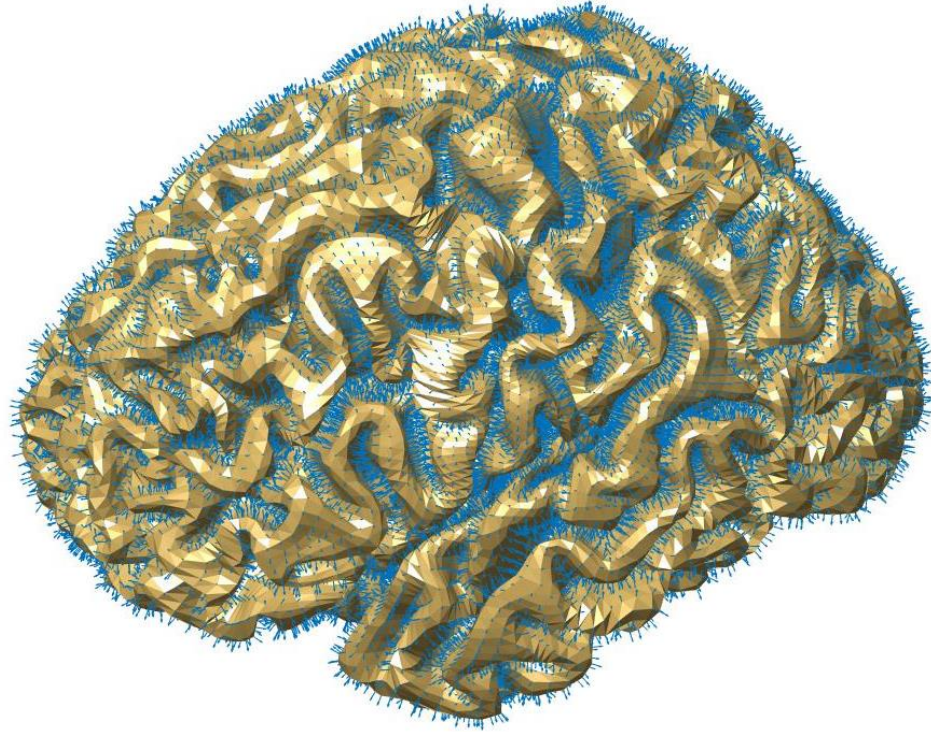
More “unknowns” than “knowns”

Infinite number of solutions can explain the data perfectly

Additional constraints required

Linear estimation problem

Distributed source model



Distributed source model: linear estimation

$$Y = G_1 q_1 + G_2 q_2 + \dots = \begin{bmatrix} G_{1,1} & G_{2,1} & \cdots \\ G_{1,2} & G_{2,2} & \cdots \\ \vdots & \vdots & \ddots \\ G_{1,N} & G_{2,N} & \cdots \end{bmatrix} \cdot \begin{bmatrix} q_1 \\ q_2 \\ \vdots \end{bmatrix} = \mathbf{G} \cdot \vec{q}$$

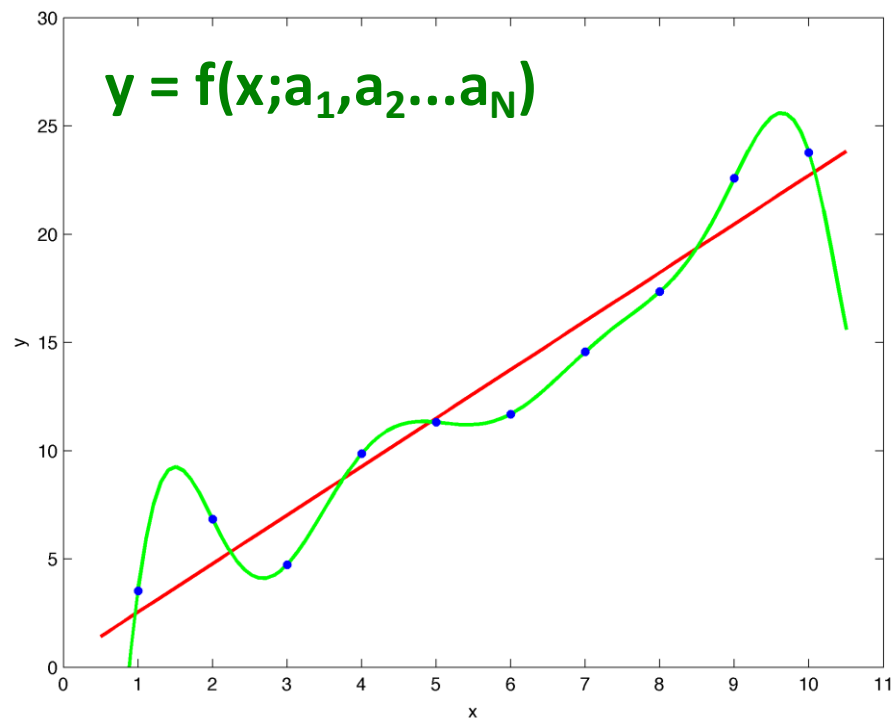
$$\vec{q} = \mathbf{G}^{-1} \cdot Y$$

Distributed source model: linear estimation

distributed source model with **many dipoles** throughout the whole brain

estimate the strength of all dipoles

data and noise can be perfectly explained



Distributed source model:
minimum-norm and regularization

$$V = G \square q + \textit{Noise}$$

$$\min_q \{ \| V - G \square q \|^2 \} = 0 \quad !!$$

Regularized linear estimation:

$$\square \min_q \{ \underbrace{\| V - G \square q \|^2}_{\text{mismatch with data}} + \lambda \underbrace{\| D \square q \|^2}_{\text{mismatch with prior assumptions}} \}$$

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Spatial filtering with beamforming

Position of the source is **not estimated** as such

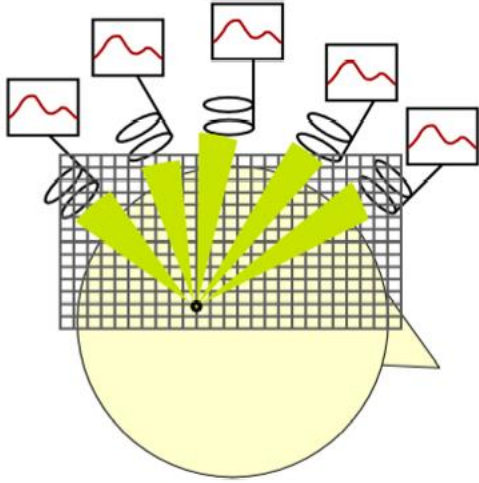
Loop over a pre-defined grid

Manipulate filter properties, not source properties

No explicit assumptions about source constraints
(implicit: single dipole)

Assumption that sources that contribute to the data are
be uncorrelated

Beamformer spatial filtering



$$Y = G_1 X_1 + G_2 X_2 + \dots + G_n X_n + \text{noise}$$

any number of n

$$Y = (G_1 X_1 + G_2 X_2 + \dots) + G_n X_n + (\text{noise})$$

$$Y = G_n X_n + N$$

N = all activity not coming from n ,
assume that N is uncorrelated with X_n

$$X'_n = W_n Y, \quad \text{where} \quad W^T = [G_n^T C_Y^{-1} G_n]^{-1} G_n^T C_Y^{-1}$$

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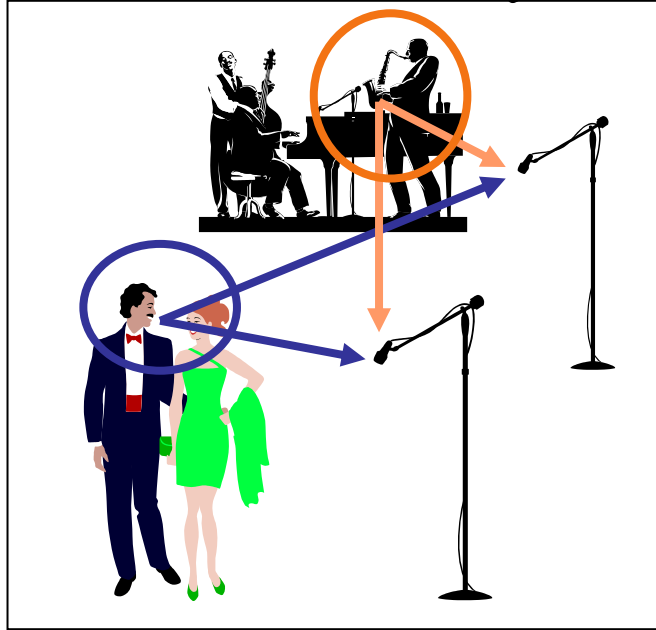
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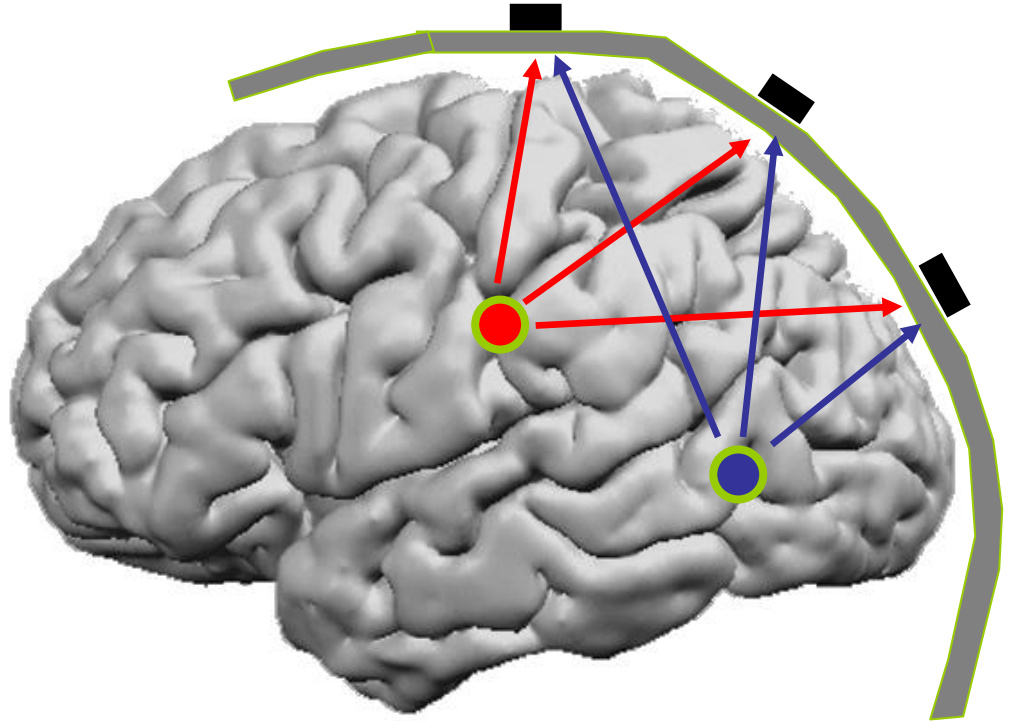
Summary

Independent component analysis

Cocktail Party



Mixture of Brain source activity



Estimating source timecourse activity using independent component analysis

$$Y = G_1X_1 + G_2X_2 + \dots + G_nX_n + \text{noise}$$

n typically the same as the number of channels

$$Y = G (X + \text{noise})$$

*includes line-noise, EOG, ECG and other
noise that is visible on all channels*

$X' = W Y$, where W maximizes the independence of X'

rows of W^{-1} correspond to $G_1, G_2, \dots$

Source modelling of independent components

Components have (maximal) independent timecourses

Assumption: components correspond to compact spatial patches (or bilateral patches)

Use simple biophysical dipole models to model the spatial component topographies

It can be challenging to match ICA sources over subjects

Summary

Forward modelling

- Required for the interpretation of scalp topographies

- Different methods with varying accuracy

Inverse modelling

- Estimate source location and timecourse from data

Assumptions on source locations

- Single or multiple point-like source

- Distributed source

Assumptions on source timecourse

- Uncorrelated (and dipolar)

- Independent

Rausschmeisser

Anything to keep in mind when applying
all this to OPM data?



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Donders Institute, Radboud University, Nijmegen, NL

janmathijs.schoffelen@donders.ru.nl